Bull. Disas. Prev. Res. Inst. Kyoto Univ., Vol. 18, Part 3, No. 141, February, 1969

Studies on the Sand Transport in Streams with Tracers

By Katsumasa YANO, Yoshito TSUCHIYA
and Masanori MICHIUE

(Manuscript received December 3, 1968)

Abstract

The purpose of this paper is to discover the mechanism of transport of uniform and graded sand and gravel in a stream with the aid of an approach using stochastic models. The irregular and intermittent movement of sediment can be expressed by a stochastic model and the stochastic characteristics of sediment transport are made clear by experimental investigations using colored sand as tracers. Moreover, it was clarified that the tracer technique is an available method for observations of the rate of transport of sand and gravel in an alluvial stream.

1. Introduction

Problems concerning the transport of solid particles in a fluid motion have attracted the attention of many investigators and have been studied in various fields. For example, the transport of sand grains by wind in a desert, sand and gravel in a stream and powder in a pipe flow has been investigated from various viewpoints. However, no theory to formulate inclusively all of these phenomena has yet been established. The mechanics of the movement of sand grains in a stream does not seem to be established theoretically, and almost all of the results describing these phenomena are given by means of experimental or dimensional analysis, because the measurement of these movements is very difficult compared with that of the movement in desert sand.

The movement of sand and gravel in streams is usually classified as bed load and suspended load. It is one of the purposes of this paper to clarify the movement process of sand and gravel in streams as bed load.

For formulae to estimate the bed load discharge, there is the well-known bed load function proposed by Einstein[1] in 1950 which is one of formulae used in many countries. In Japan, there are some formulae to estimate the bed load discharge, such as the formula introduced by Sato, Kiikawa and Ashida[2] in 1957, based on a concept similar to Einstein's one. On the other hand, Kalinske[3] established in 1947, a theory of the movement of sand grains in a stream, based on a concept different from Einstein's one, and proposed a formula to estimate the bed load discharge. Recently, it has been assumed that the movement of a single sand grain in a stream is similar to that of one in desert sand, and the velocity, height and distance of the saltation of a sand grain have been calculated by Yalin[4] in 1963, Kishi and Fukuoka[5] in 1966, and Tsuchiya, Watado and Aoyama[6] in 1968, respectively. Moreover, some equations for estimating bed load discharge were introduced by Yalin and by Kishi and Fukuoka respectively, based on these calculations and experimental results.

Although the sediment discharge is usually expressed by a function of the flow intensity, the movement mechanism of sand grains is not completely clarified yet and these formulae are dynamically insufficient. The present situation is that a universal theory of sand movement has not been established yet. In order to clarify the mechanics of such complex sand movement in a stream, methods of measuring the movement of a single sand particle in a flow and researching the characteristics of sand movement as a group must be considered, since the dynamics of sand movement seems to be established when both of these approaches are combined. In this paper, the authors intend to investigate the mechanics of sand movement as a group.

Sand grains start to move by hydrodynamic drag forces acting on a sand grain caused by mean velocity and turbulence of flow and collision of moving sand grains with bed ones, but they usually rest after moving a certain distance. The movement is a repeat phenomenon of irregular and intermittent motion. When we trace the motion of a sand grain by a Lagrangian method, it is not generally a deterministic motion, but an undeterministic one. In order to describe the movement of sand grains which is an undeterministic motion, it seems necessary to introduce a stochastic concept. Hubbell and Sayre[7,8] recently proposed a method of measuring the bed load discharge in an alluvial stream with radioisotope tracers, and mathematically formulated the movement of sand grains as stochastic processes by an attractive method. The authors performed detailed experiments with colored sand and gravel as tracers in a laboratory flume to discover the mechanism of movement of uniform and graded sand and gravel, based on a concept similar to Hubbell and Sayre's. Moreover, in this paper, a method for estimating the sediment discharge is considered on the basis of these experimental results.

2. A stochastic model of sand movement

Let us consider the case where the discharge of flow and sediment is constant in space and time and the transport of sediment can be considered as bed load. A colored sand grain having the same characteristics of transport as bed sediment begins to move from the origin $x=0$ at the time $t=0$. As regards the repeat phenomenon of irregular and intermittent motion, the movement of a colored sand grain can be described with the aid of an approach employing stochastic processes introducing the concept of a step defined by Einstein in it. Such a single step is defined as a process in which the movement of sand grains begins and then rests again after it has begun. Let us make the assumptions that the physical process of movement of colored sand grains is homogeneous in space and time and those changes in the future are independent of past changes. This means that the probability of taking such a step is dependent only on the interval distance and is independent of the location and past movement of the sand grains. From this point of view we can formulate the movement of colored sand grains mathematically.

If the probabilities of taking more than one step between a certain infinitesimal interval $(x, x+h)$ can be expressed by $\lambda_1 h+o(h)$, the probability of taking no step between the interval is expressed by $1-\lambda_1 h-o(h)$, in which λ_1 is a positive probability constant defining the movement probability per unit distance, and the term $o(h)$ denotes a quantity of a smaller order of magnitude than h. When a colored sand grain moves between the interval $(0, x+h)$, the differential equation to formulate the probability $p(n;\ x+h)$ defining the changes of n steps can easily be written as[9]

$$dp(n;\ x)/dx = -\lambda_1 p(n;\ x) + \lambda_1 p(n-1;\ x) \quad (n \geqq 1) \qquad \cdots\cdots(2.1)$$

And for the condition that $n=0$,

$$dp(0;\ x)/dx = -\lambda_1 p(0;\ x) \qquad \cdots\cdots(2.2)$$

By using the initial conditions that $p(0;\ 0)=1$ and $p(n;\ 0)=0$, the integration of Eqs. (2.1) and (2.2) yields the following Poisson distribution.

$$p(n;\ x) = e^{-\lambda_1 x}(\lambda_1 x)^n/n! \quad (n=0,\ 1,2,\cdots) \qquad \cdots\cdots(2.3)$$

The probability that a colored sand grain still exists at the distance x after it has taken n steps, can be expressed by $1-\sum_{i=0}^{n-1} e^{-\lambda_1 x}(\lambda_1 x)^i/i!$. Since this denotes the distribution function for the location of the colored sand grain after taking n steps, the differentiation of this with respect to x yields the probability density function expressed as

$$f(x;\ n) = \lambda_1 e^{-\lambda_1 x}(\lambda_1 x)^{n-1}/\Gamma(n) \qquad \cdots\cdots(2.4)$$

in which $f(x;\ n)$ is the probability density function for the location of the colored sand grain after taking $n (n \geqq 1)$ steps, and $\Gamma(n)$ denotes the Gamma function. Substituting $n=1$ into Eq. (2.4), the probability density function for the traveling distance of the colored sand grain taking a single step can be expressed as

$$f(x;\ 1) = \lambda_1 e^{-\lambda_1 x} \qquad \cdots\cdots(2.5)$$

It is found from Eq. (2.5) that the traveling distance of the colored sand grain after taking a single step can be expressed by an exponential distribution. This seems to be a very interesting phenomenon. Multiplying Eq. (2.4) by x, the integration of this yields the average traveling distance of a colored sand grain taking n steps downstream,

$$\int_0^\infty x f(x;\ n)\,dx = n/\lambda_1$$

Therefore, the average traveling distance of a colored sand grain taking a single step, defined by Einstein can be expressed by $1/\lambda_1$.

Now consider the movement of a colored sand grain in time. Defining the step with respect to time by the process during the time when the colored sand grain begins to rest and then begins to move and finishes moving, it can be expressed by the same step as a single one for the traveling distance. When a colored sand grain travels during the time interval $(0, t)$, the probability of occurance $p(n;\ t)$ defining changes of n steps can be described by a similar equation to Eq. (2.3) as follows;

$$p(n;\ t) = e^{-\lambda_2 t}(\lambda_2 t)^n/n! \qquad \cdots\cdots(2.6)$$

in which λ_2 is a positive probability constant, expressing the movement probability per unit time. Substituting $n=0$ into Eq. (2.6), the probability $p(0, t)$ for the colored sand grain staying still at the origin can be written as:

$$p(0\,;\,t)=e^{-\lambda_2 t} \qquad \cdots\cdots(2.7)$$

When a colored sand grain travels downstream, the relation between the time and traveling distances is shown schematically in Fig. 1. Let the colored sand grain begin to take steps in time. Since the colored sand grain having tried to take n steps in distance takes n steps in time, the probability density function with respect to the distance of the colored sand grain having taken n steps at time t can be written as

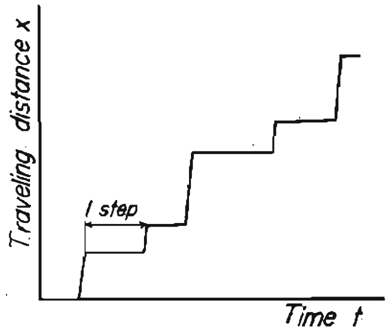

Fig. 1. Schematic diagram of sand movement.

$$f_t(x\,;\,n)=f(x\,;\,n)p(n\,;\,t) \qquad \cdots\cdots(2.8)$$

in which $f_t(x\,;\,n)$ denotes the probability density function with respect to the distance of the colored sand grain having taken n steps at time t. Since, therefore, the summation of the probability density function of all the steps equals the probability density function of the colored sand grain in time and space, the function becomes[7]

$$f_t(x)=\sum_{n=0}^{\infty}f(x\,;\,n)p(n\,;\,t)$$
$$=\lambda_1 e^{(-\lambda_1 x+\lambda_2 t)}(\lambda_2 t/\lambda_1 x)^{1/2}I_1(2\sqrt{\lambda_1 x\lambda_2 t}\,) \qquad \cdots\cdots(2.9)$$

in which $I_1(2\sqrt{\lambda_1 x\lambda_2 t}\,)$ is the modified Bessel function of the first order. The average traveling distance $\bar{x}$ and the variance of the function σ^2 are expressed respectively as:

$$\bar{x}=\int_0^{\infty}xf_t(x)dx=(\lambda_2/\lambda_1)t \qquad \cdots\cdots(2.10)$$

$$\sigma^2=\int_0^{\infty}(x-\bar{x})^2 f_t(x)dx=2\lambda_2 t/\lambda_1^2-(\lambda_2 t)^2 e^{-\lambda_2 t}/\lambda_1^2 \fallingdotseq 2\lambda_2 t/\lambda_1^2 \qquad \cdots\cdots(2.11)$$

3. Experiments on the transportation of uniform sand and gravel

3. 1 *Experimental procedure*

The experiments were conducted in a laboratory flume of rectangular cross-section 20 cm deep and 14 cm long, the slope of the flume being adjustable. The depth of sand was about 5 cm and the bed slope was adjusted 0.01. Colored sand grains having the same size as that of bed sediments were set at the origin. Longitudinal distributions and variations of numbers of the colored sand grains existing at the origin were measured. The mean sizes and specific gravities of the two kinds of sand grains used were 3.5 mm and 2.65, and 6.75 mm and 1.24 respectively. The colored sand grains used as tracers were labeled with paint on the sand grains. The numbers of colored sand grains set at the origin were from 100 to 200, and the depth of the colored sand grains was of the particle size of a sand grain. These experiments were conducted in the regime of so-called transition and flat beds. Therefore, bed roughness had little influenced on the phenomena of the movement of sand grains. The conditions of the experiments performed are summarized in Table 1.

TABLE 1.
Conditions of experiment performed.

Run No.	Water depth h (cm)	Slope I	Shear velocity u_* (cm/sec)	Specific gravity	Diameter d (cm)
1 a	2.54	1.00×10^{-2}	4.99	2.65	0.35
1 b	2.54	1.00	4.99	〃	〃
2 a	3.62	1.01	5.98	〃	〃
2 b	3.56	1.01	5.94	〃	〃
2 c	3.51	0.961	5.76	〃	〃
3 a	4.30	1.01	6.45	〃	〃
3 b	4.25	1.03	6.55	〃	〃
4 a	3.37	1.04	5.86	〃	〃
4 b	3.22	1.02	5.67	〃	〃
4 c	3.16	1.00	5.56	〃	〃
5 a	4.00	0.99	6.24	〃	〃
5 b	3.81	1.05	6.27	〃	〃
6 a	2.86	1.00	5.30	〃	〃
7 a	2.70	1.00	5.14	〃	〃
8 a	3.45	1.04	5.93	〃	〃
8 b	3.25	1.02	5.70	〃	〃
8 c	3.10	0.94	5.77	〃	〃
9 a	3.30	0.97	5.60	〃	〃
9 b	3.44	0.98	5.75	〃	〃
10 a	5.45	1.03	7.42	〃	〃
11 a	3.72	1.00	6.03	〃	〃
12 a	42.8	1.04	6.61	〃	〃
1	3.90	0.90×10^{-2}	5.87	1.24	0.675
2	3.30	0.97	5.60	〃	〃
3	5.00	1.05	7.17	〃	〃
4	5.37	1.10	7.60	〃	〃
5	2.87	0.98	5.26	〃	〃
6	5.87	1.05	7.78	〃	〃

3. 2 Experimental results and considerations

(1) Adaptability of the stochastic model to sand movement

One example of the experimental results of the probability for colored sand grains staying still at the origin is shown in Fig. 2. In order to ascertain the reappearance of the experiments, the experiments for the colored sand grains set at distances of 7 m and 8 m from the downstream end of the flume were conducted, and one of the experimental results is shown in the figure. It is seen that both the results are in good agreement with each other, and that therefore the reappearance of the experiments is very good. The experimental values of the probability of staying still at the origin are in good agreement with an exponential function expressed by Eq. (2. 7). However, the experimental values at $t=0$ are different from the value, unity, given by Eq. (2. 7). This fact may be due to the experimental procedure for setting colored sand grains at the origin. Supposing that the influence of this initial condition can be eliminated, therefore, the experimental values are in good agreement with Eq. (2. 7).

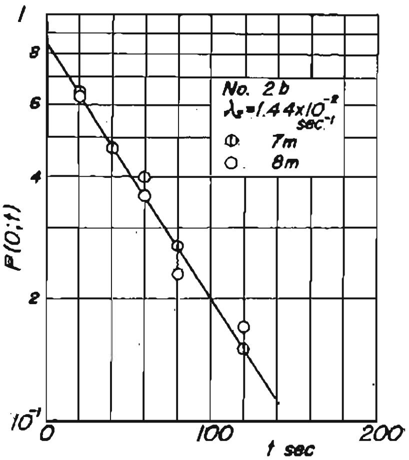

Fig. 2. The variation of probability of sediment staying still at the origin.

Examples of the comparisons between the distributions of traveling distance for a single step, obtained theoretically by the stochastic model and the experimental values are shown in Fig. 3. The theoretical curves expressed by Eq. (2. 5) are shown in this figure. It is found that both the experimental values and the theoretical curves are expressed by an exponential distribution.

It was confirmed, moreover, from the results that the difference between both the comparisons is significant at the five percent level by the x^2 test, and that the distribution of traveling distance of a single step can be expressed by an exponential function.

An experimental result of the time variation in the average traveling distance of all the colored sand grains is shown in Fig. 4. It is found that the experimental values shown in the figure are in good agreement with the theoretical

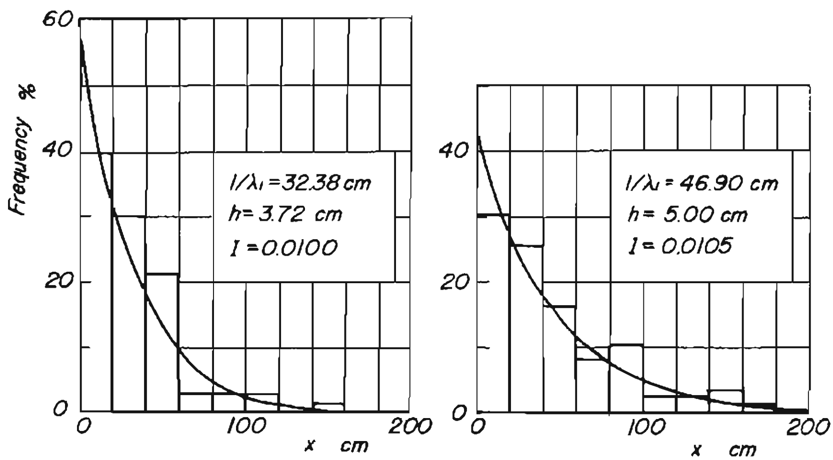

Fig. 3. Distribution of traveling distance of a single step.

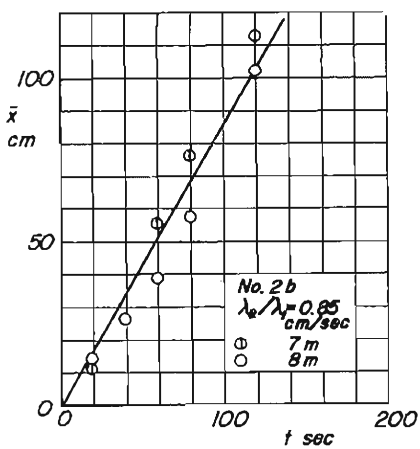

Fig. 4. An example of change of average traveling distance of tracers with time.

line expressed by Eq. (2.10) because the average traveling distance is proportional to time. Therefore, the average traveling velocity of all the tracers can be determined by the slope of a straight line fitting the experimental values.

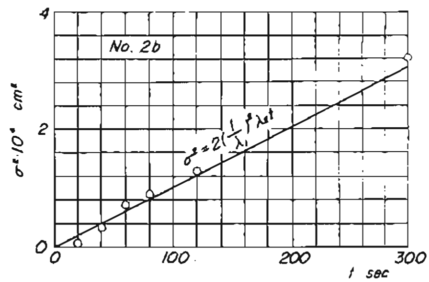

Fig. 5. An example of time changes of variance of distribution.

The time change of the variance of distribution of colored sand grains is shown in Fig. 5. The variance changes linearly with time, and the experimental values are in good agreement with the theoretical line expressed by Eq. (2.11). Therefore, two probability constants λ_1 and λ_2 can be estimated using Figs. 2 and 4. The values λ_1 and λ_2 should be estimated using Figs. 4 and 5, because the experimental values shown in Fig. 2 seem to show a slight influence of the initial condition for setting colored sand grains. Substituting these two probability constants λ_1 and λ_2 estimated from the experimental results into Eq. (2.9), an example of the longitudinal distributions of colored sand

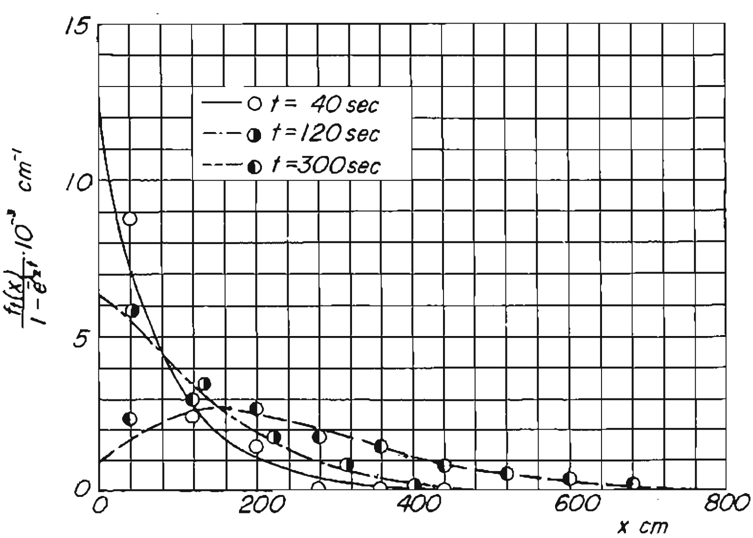

Fig. 6. Longitudinal distributions of colored sands at various times.

grains at various times is shown in Fig. 6. It is found from the figure that in the dispersive processes of colored sand grains set at the origin, the longitudinal distribution is expressed by an exponential function at an early stage, but after a few minutes it becomes a distribution with a maximum which decreases with time. It was confirmed that the difference between the theoretical curves and the experimental values is significant at the five percent level by the χ^2 test. It is concluded from the experimental results that the stochastic model proposed can describe the movement of sand grains in a stream.

(2) Hydraulic characteristics of probability constants in the stochastic model

Relations between the probability constants in the stochastic model and the hydraulic characteristics are investigated here. The experimental results on the probability constants are shown in Table 2. Fig. 7 shows a plot of the dimensionless average traveling distance of a single step with a parameter of

TABLE 2.
Results of experiment for stochastic characters and rate of sediment transport.

Run No.	$1/\lambda_1 d$	λ_2/λ_1 cm/sec	$\lambda_2\{d/(\sigma/\rho-1)g\}^{1/2}$	q_B cm²/sec	$1/\psi$
1 a	1.94×10^2	1.33×10^{-2}	2.88×10^{-6}	3×10^{-4}	4.40×10^{-2}
1 b	0.98	5.67	24.3	5	4.40
2 a	1.18	64.1	230	280	6.33
2 b	1.40	49.2	147	180	6.23
2 c	2.42	53.2	92.4	380	5.84
3 a	1.53	128	352	—	7.35
3 b	1.95	76	163	—	7.58
4 a	0.95	42.6	188	—	6.07
4 b	1.37	32	98.4	—	5.69
4 c	2.47	31.2	53.1	230	5.47
5 a	1.76	61	146	660	6.88
5 b	1.94	35.5	76.9	710	6.93
6 a	4.34	12.5	12.1	10	4.95
7 a	1.56	3.67	9.9	12	4.68
8 a	1.98	52	110	350	6.21
8 b	1.64	58	149	270	5.74
8 c	0.76	71.9	397	150	5.05
9 a	2.04	84	174	380	5.54
9 b	2.14	60	118	390	5.84
10 a	1.63	322	831	3510	9.72
11 a	2.26	144	268	660	6.44
12 a	1.25	256	859	890	7.71
1	1.47×10^2	8.54	46×10^{-4}	0.54	2.17×10^{-1}
2	3.67	4.20	9.1	0.94	1.97
3	2.41	6.13	20.1	1.96	3.24
4	1.93	7.30	30.1	2.65	3.65
5	3.41	4.90	11.4	0.37	1.74
6	2.92	7.33	19.9	2.61	3.80

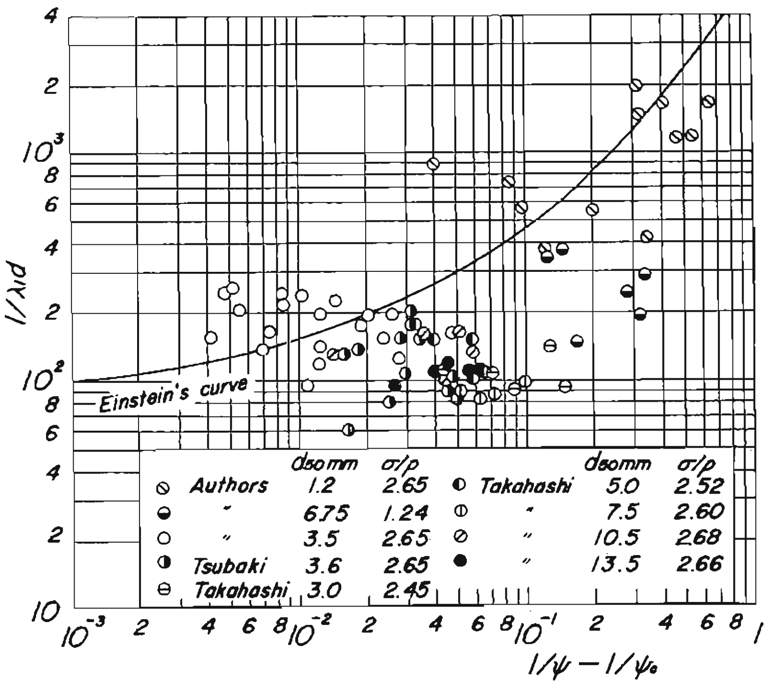

Fig. 7. Relation between dimensionless average traveling distance of a singe step and flow intensity.

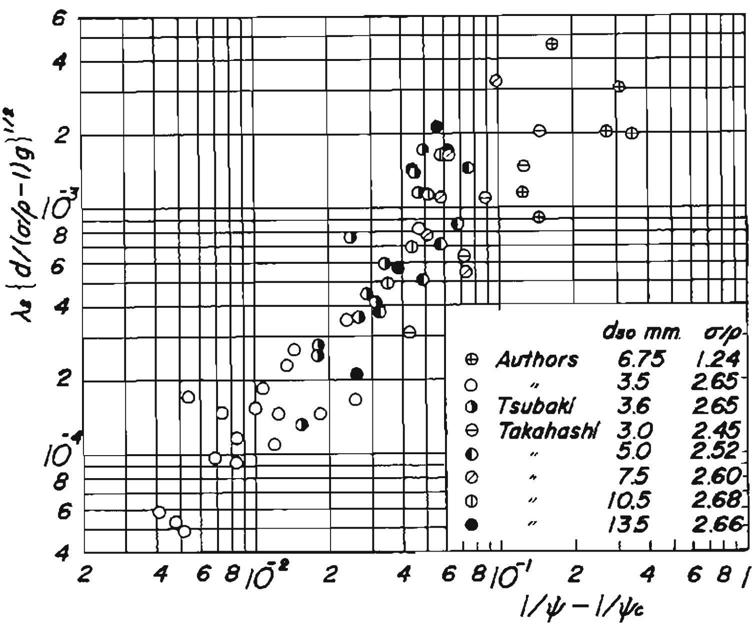

Fig. 8. Relation between dimensionless probability for the beginning of movement of sand grains per second and flow intensity.

the flow intensity defined by $1/\psi - 1/\psi_c = (u^2_* - u_{*c}^2)/[\{(\sigma/\rho)-1\}gd]$, in which σ and ρ are the density of sand grains and fluid, respectively, g the acceleration of gravity, d the diameter of grains, and u_* the shear velocity. The subscript, c, refers to the initial threshold conditions of the grain movement. Moreover, the curves shown in this figure describe the theoretical average traveling distance of a single step proposed by Einstein[1] and the experimental values obtained by Shinohara and Tsubaki[10], and Takahashi[11] respectively are shown for comparison. It is seen from the results that the average traveling distance of a single step is nearly constant and about 80 to 250 times of grain size within the range of flow intensity less than 0.1. In the range of flow intensity larger than 0.1, the average traveling distance of a single step increases with the increase of the value of flow intensity. Moreover, Einstein's curve is in good agreement with the experimental values, although the values scatter much.

Probabilities for the beginning of movement of sand grains per second shown in Fig. 8 increase sensitively with the increase of the value of flow intensity, and are very different from the change of average traveling distance of a single step shown in Fig. 7. The increase of the rate of bed load with the increase of the value of flow intensity seems to be independent of the change of average traveling distance of a single step, and is mainly dependent on the increase of the probability for the beginning of movement of sand grains. It is considered therefore that the rest period of sand grains generally decreases with the

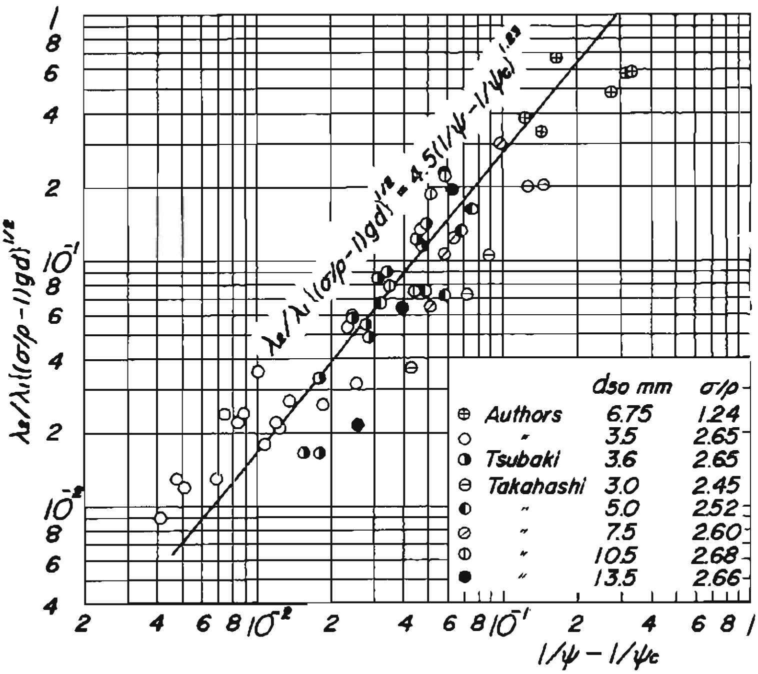

Fig. 9. Relation between the dimensionless average velocity of movement of sand grains and flow intensity.

increase of the value of flow intensity and that the transport mechanism of sand grains will change from a bed load to a suspended one in the case where the value of flow intensity becomes very large.

Fig. 9 shows a relation between the dimensionless average velocity of the movement of sand grains and the flow intensity. Although the experimental values scatter much, an empirical formula shown in the figure seems to be proposed.

Using this relation, an empirical formula for the estimation of the bed load discharge with colored sand grains as tracers can be proposed below. Since the average traveling velocity of sand grains λ_2/λ_1 is formulated as Eq. (2. 10), the bed load discharge can be estimated as the value multiplied the volume of sand grains in the movement depth of ones by the velocity. Although the estimation of the movement depth is very difficult, we can assume that the movement depth is nearly equal to one grain size within the regime of the experiments carried out. The bed load discharge can be expressed as

$$q_B=(1/k_1d^2)k_2d^3(\lambda_2/\lambda_1) \qquad \cdots\cdots\cdots\cdots(3.1)$$

in which q_B is the bed load discharge per unit width, and both k_1 and k_2 are constants estimated from the shape of sand grains. Using the experimental value of k_2/k_1 which is estimated as 0.4, Eq. (3. 1) can be rewritten as

$$q_{B*}\equiv q_B/u_*d=(0.4\lambda_2)/(\lambda_1 u_*) \qquad \cdots\cdots\cdots\cdots(3.2)$$

A comparison between the line calculated by Eq. (3. 2) and experimental values is shown in Fig. 10. The experimental values of the rate of bed load are a little less than the calculated ones within the small value range of $\lambda_2/\lambda_1 u_*$, but the experimental values are in good agreement with the calculated line

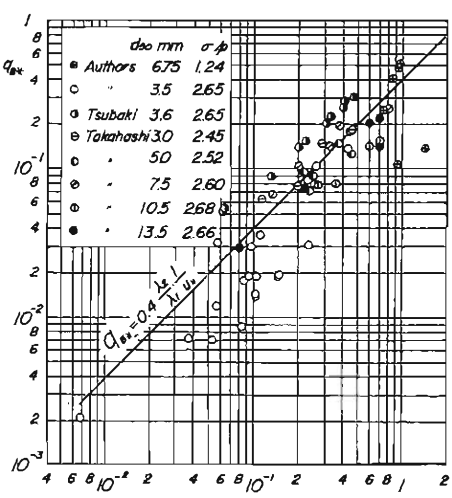

Fig. 10. Relation between dimensionless rate of sediment transport q_{B*} and value of $\lambda_2/\lambda_1 u_*$.

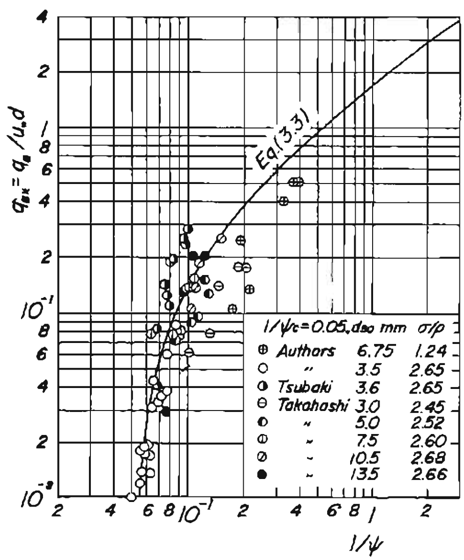

Fig. 11. Comparison between curve calculated by an empirical formula proposed and experimental values.

as a whole.

Supposing that the average traveling velocity of sand grains can be described by the empirical formula shown in Fig. 9, an empirical formula for estimating the rate of bed load can be expressed as

$$q_{B*} = 1.8\psi^{1/2}(1/\psi - 1/\psi_c)^{1.23} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(3.3)$$

Fig. 11 shows the comparison between the curve caluculated by Eq. (3. 3) and the experimental values.

It is concluded from these results that the rate of bed load is in good agreement with Eq. (3. 3) having two probability constants which are estimated by the method of tracers. Since we assumed that the movement depth is one grain size below the bed surface in deriving Eq. (3. 2), the formula will no longer be sufficient to estimate the bed load discharge when the flow intensity is very large. Although we intend to investigate these problems based on further detailed expriments, it was found, within the regime of the experiments shown in Fig. 11, that the method of tracers is appliciable to find a method for estimating the rate of bed load.

4. Experiments on transportation of graded sand and gravel

4. 1 *Experimental procedure*

Transport characteristics of graded sand and gravel are investigated by means of tracers in this chapter. Some experiments on graded sand and gravel were conducted by the same method as those for uniform sand and gravel. These experiments were perfoımed in an adjustable-slope and recirculating flume of rectangular crossection, 20 cm wide, 20 cm deep and 20 m long. The depth of sand grains was about 5 cm, and the bed slope was adjusted 0.026. The sand and gravel used were two kinds of materials, 3.5 mm in median diameter, which consisted of the mixture of sand and gravel of A and B, and A, B and C respectively. Sieve analysis curves of sand and gravel of A, B and C are shown in Fig. 12. Probability constants and bed load discharge of elementary sand and gravel were measured.

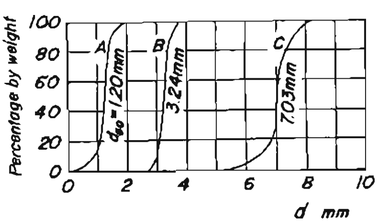

Fig. 12. Sieve analysis curves of sand and gravel used.

4. 2 *Experimental results and considerations*

(1) Hydraulic characteristics of probability constants in the case of graded sand and gravel

An example of the longitudinal distributions of graded sand and gravel as tracers is shown in Fig. 13. The curves in this figure are calculated by Eq. (2. 9) after being normalized. The experimental values scatter due to the effects of bed roughness and separation of bed materials, but these characteristics are in good agreement with the theoretical curves. We can understand from the figure that the average traveling velocities of large gravels in graded sediments are larger than those of small ones, when all the bed materials are moving.

Relations between the average traveling velocity of graded sand and gravel and the flow intensity are considered here. Fig. 14 shows a relation between the

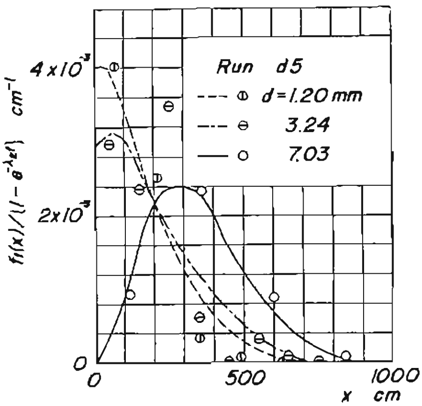

Fig. 13. Longitudinal distributions of various sand grains after 10 seconds.

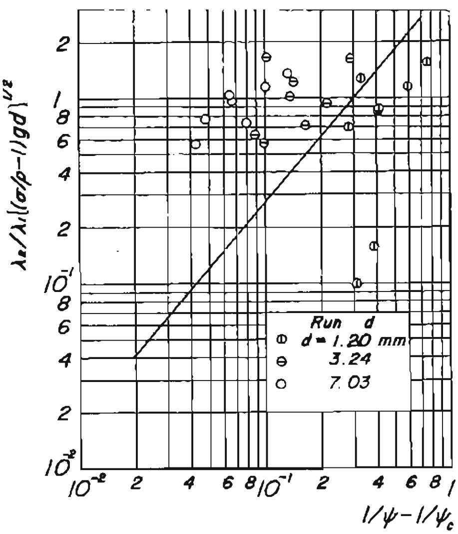

Fig. 14. Relation between dimensionless average traveling velocity of graded sand grains and flow intensity.

average traveling velocity of the elementary sand and gravel in the case of mixtures of three kinds of sediments and the flow intensity. A straight line in the figure shows an empirical formula already shown in Fig. 9. The average traveling velocity of the elementary sand and gravel is very different from that of uniform sand grains, and seems to be independent of the flow intensity. It is found from the figure that almost all small sand grains are affected by the hiding effect of the grains and some of these are affected by the accelerating effect of the grains. Such a phenomenon is considered to be due to the irregularity of bed surfaces consisting of graded sand and gravel. This fact suggests that the irregularity of bed surfaces greatly affects the movement of sand and gravel.

Relations between the flow intensity and the probabilyty constants, λ_1 and λ_2, are considered below. Fig. 15 shows a plot of the dimensionless average traveling distance of a single step versus the flow intensity. Although experimental values are very scattered, it is found that the average traveling distance of a single step for graded sand and gravel is smaller than that of uniform sand and gravel. The probability for the beginning of movement of large sand grains is larger than that of small sand grains, and this tendency is very different from that of uniform sand and gravel shown in Fig. 8 (see Fig. 16). Qualitative transport characteristics of graded sand and gravel are made clear by this, but it seems to be very difficult to discuss quantitative transport characteristics. We intend to investigate the transport mechanism of sand and gravel with tracers in the future.

(2) Rate of transport of graded sand and gravel

The bed load discharge of elementary sand and gravel is shown in Fig. 17. The curve in this figure indicates Einstein's formula of the rate of transport of uniform sand grains. Experimental values show that the rate of transport of

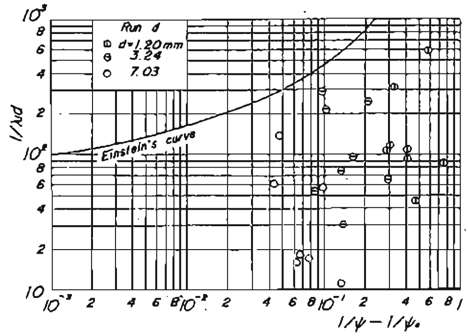

Fig. 15. Relation between dimensionless average traveling distance of a single step and flow intensity in the case of graded sediment.

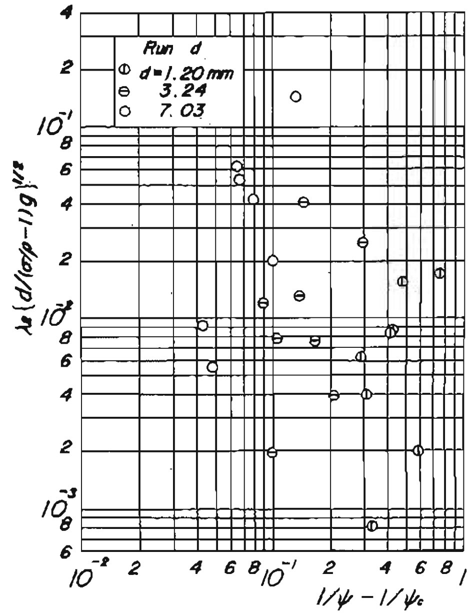

Fig. 16. Relation between dimensionless probability for the beginning of movement of graded sediment and flow intensity.

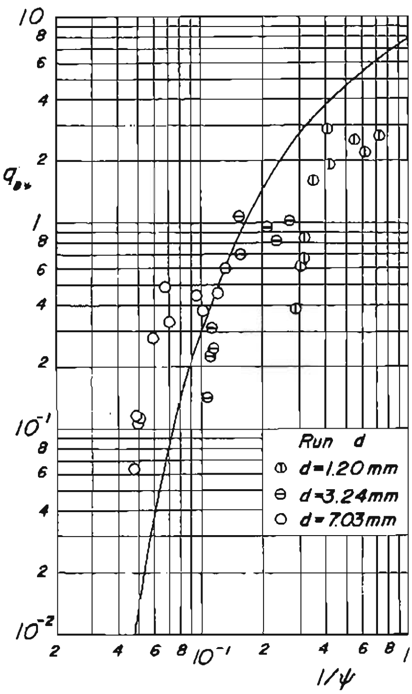

Fig. 17. Comparison between experimental values of rate of transport of elementary sand and gravel and Einstein's formula of rate of transport of uniform sediment.

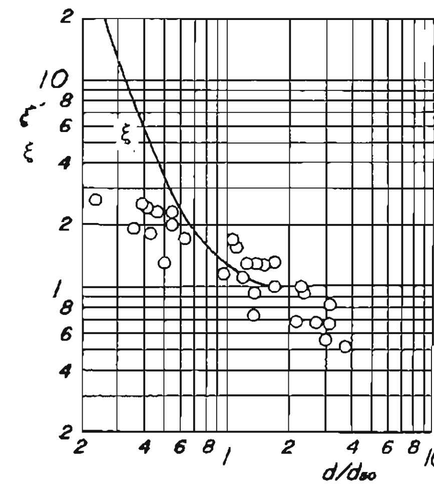

Fig. 18. Comparison between the hiding factor of grains proposed by Einstein and ξ'.

large sand grains becomes larger than that of uniform sand grains due to the accelerating effect of the grains, and that the rate of transport of small sand grains is less than that of uniform sand grains because of the hiding effect of the grains. These facts agree with the results of the experiments performed with tracers. The ratio of the theoretical value of Einstein's formula to the measured rate of transport of elementary sand and gravel is defined as ξ'. Fig. 18 shows the changes in the ratio obtained from various kinds of experiments with graded sand and gravel. The hiding factor of grains ξ proposed by Einstein[1] is very large compared with the experimental results. It is clarified from the figure that the value of ξ' becomes smaller than unity because of the accelerating effect of large sand grains in the case of $d/d_{50}>1.5$. The irregularity of bed surfaces of graded sand and gravel is shown to play an important role concerning the rate of sand transport. The movement depth of sand and gravel should be investigated to estimate the rate of transport of graded sand and gravel with tracers in the future.

5. Conclusion

The results obtained can be summarized as follows. It was concluded from the experiments performed with tracers that the stochastic model can simulate the movement of sand and gravel. Using the stochastic model, the distribution of the traveling distance of a single step of sand grains can be described by an exponential function, and this result is in good agreement with the experimental values.

The average traveling distance of a gingle step is nearly constant and about 80 to 250 times of grain size in the regime of which values of flow intensity are less than 0.1, and it increases with the increase of the values of flow intensity in the regime of which the values are larger than 0.1. On the other hand, the probability for the beginning of movement of sand grains per second sensitively increases with the increase of the flow intensity. Therefore, in the regime of which the values of flow intensity are small the increase of bed load discharge is independent of the changes in average traveling distances of a single step, and is mainly dependent on the increase of the probability for the beginning of movement of sand grains.

An empirical formula to express the relation between the average traveling velocity of sand grains and the flow intensity was proposed on the basis of the experimental results conducted with tracers. Based on an assumption that the movement depth of sand grains is nearly equal to one grain size below the bed surface, an empirical formula for estimating the rate of transport of sand grains was proposed. Since the formula is in good agreement with the experimental values in the regime of the experiment carried out, the authors found that a method of tracers is available for estimating the bed load discharge in streams.

The average traveling velocity of large sand grains in graded sediments is larger than that of uniform sediments due to the accelerating effects of the grains, and on the other hand, the velocity of small sand grains is smaller than that of uniform ones due to the hiding effects of the grains. It was found that there is also such a phenomenon in measuring the rate of transport of sand grains in graded sediments.

Acknowledgements

The authors wish to thank Messrs. T. Aoyama and F. Yoshikawa for their assistance in the experiments and computation. Thanks are also due to Mr. M. Sumino, Research Assistant, for his help in preparing the paper and Miss M. Motomochi for her help in typing the manuscript.

References

1) Einstein, H. S.: The Bed-Load Function for Sediment Transportation in Open Channel Flow, U. S. Department of Agriculture, Soil Conservation Service, Tech. Bull. No. 1025, 1950, pp. 1-71.
2) Sato, S., Kikkawa, H. and Ashida, K.: Research on the Bed Load Transportation, Jour. of Res., Pub. Works Res. Inst., No. 98, 1958, pp. 1-71 (in Japanese).
3) Kalinske, A. A.: Movement of Sediment as Bed Load in Rivers, Trans. A. G. U., Vol. 28, No. 4, 1947, pp. 615-620.
4) Yalin, M. S.: An Expression for Bed-Load Transportation, Proc. A. S. C. E., Vol. 89, HY3, 1963, pp. 221-250.
5) Kishi, T. and Fukuoka, S.: The Mechanism of Saltation of Sand Particles and Bed Load Transportation, Tech. Report, Civil Eng. Soci. in Hokkaido, No. 23, 1966, pp. 113-120.
6) Tsuchiya, Y., Watado, K. and Aoyama, T.: On the Mechanism of Saltation of a Sand Particle in a Turbulent Stream (1), Annuals, D. P. R. I., Kyoto Univ., No. 12, 1968 (in press, in Japanese).
7) Sayre, W. W. and Hubbell, D. W.: Transport and Dispersion of Labeled Bed Material, North Loup River, Nebraska, U. S. Geo. Survey, Water Resources, 1963.
8) Hubbell, D. W. and Sayre, W. W.: Sand Transport Studies with Radioactive Tracers, Proc. A. S. C. E., Vol. 90, HY3, 1964, pp. 39-68.
9) Feller, W.: An Introduction to Probability Theory and Its Applications, Modern Asia Edition, Vol. 1, 1960, p. 400.
10) Shinohara, K. and Tsubaki, T.: A Consideration on the Mechanism of Movement of Sand and Gravel, Reports, Res. Inst. for Applid Mechanics, Kyushu Univ., Vol. 10, 1957, pp. 85-94 (in Japanese).
11) Takahashi, M.: Experimental Studies on the Transport of Sand and Gravel (First Report), Shin-sabo, Jour. Erosion Control Eng., Vol. 18, No. 4, 1966, pp. 5-14 (in Japanese).